

PRECISION MEASUREMENTS, EXTRA GENERATIONS AND HEAVY NEUTRINO

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The existence of extra chiral generations with all fermions heavier than M_Z is strongly disfavored by the precision electroweak data. The exclusion of one additional generation of heavy fermions in SUSY extension of Standard Model is less forbidden if chargino and neutralino have low degenerate masses with $\Delta m \simeq 1$ GeV. However the data are fitted nicely even by a few extra generations, if one allows neutral leptons to have masses close to 50 GeV. Such heavy neutrino can be searched in the reaction $e^+e^- \rightarrow N\bar{N}\gamma$ at LEP-200 with total final luminosity of $2600pb^{-1}$.

1 Introduction

The straightforward generalization of the Standard Model (SM) through inclusion of extra chiral generation(s) of heavy fermions, quarks ($q = U, D$) and leptons ($l = N, E$), is an example of New Physics at high energies which does not decouple at “low” ($\sim m_Z$) energies. New particles contribute to physical observables through self-energies of vector and axial currents. This gives corrections ¹ δV_i to the functions $V_i(i = A, R, m)$ which determine ² the values of physical observables (axial coupling g_A , the ratio $R = g_V/g_A$, and the ratio m_W/m_Z).

We consider the case of several lepton and quarks $SU(2)_L$ doublets and their right-handed singlet companions: $(UD)_L, U_R, D_R, (NE)_L, N_R, E_R$. In what follows we will assume that the mixing among new generations and the three existing ones is small, hence new fermions contribute only to oblique corrections (vector boson self energies).

2 LEPTOP fit to experimental data

We compare theoretical predictions for the case of the presence of extra generations with

experimental data ³ with the help of the code LEPTOP ⁴. These experimental data are the latest updates presented at this conference and they are well fitted by Standard Model. We perform the four parameter $(m_t, m_H, \alpha_s, \bar{\alpha})$ fit ^a to 18 experimental observables.

The fitted parameters ^b together with the values of the predicted observables and their pulls from the experimental data are given in the Table 1. Only the experimental value of the forward-backward asymmetry in the Z decay into the pair of b-quarks A_{FB}^b shows a hint for disagreement with Standard Model. We take $m_D = 130$ GeV – the lowest value allowed for the new quark mass from Tevatron search ⁷ and take $m_U \gtrsim m_D$. As for the leptons from the extra generations, their masses are independent parameters. To simplify the analyses we start with $m_N = m_U$,

^aThe mass of Z-boson in the fit was fixed to the latest experimental value $M_Z = 91.1875(21)$ GeV

^b During this conference the new results on the electron-positron annihilation into hadrons in the range $\sqrt{s} = 2 - 5$ GeV from BES ⁵ were released. With $\bar{\alpha}^{-1} = 128.945(60)$ ⁶ recalculated using these new BES results, we get from LEPTOP fit slightly higher prediction for the higgs mass $m_H = 78_{-32}^{+53}$ GeV, $m_t = 174.1(4.5)$ GeV, $\alpha_s = 0.1182(27)$, $\bar{\alpha}^{-1} = 128.927(58)$ and $\chi^2/ndf = 21.1/14$.

Table 1. LEPTOP fit of the precision observables.

Observ.	Exper. data	LEPTOP fit	Pull
Γ_Z [GeV]	2.4952(23)	2.4964(16)	-0.5
σ_h [nb]	41.541(37)	41.479(15)	1.7
R_l	20.767(25)	20.739(18)	1.1
A_{FB}^l	0.0171(10)	0.0164(3)	0.7
A_τ	0.1439(42)	0.1480(13)	-1.0
A_e	0.1498(48)	0.1480(13)	0.4
R_b	0.2165(7)	0.2157(1)	1.2
R_c	0.1709(34)	0.1723(1)	-0.4
A_{FB}^b	0.0990(20)	0.1038(9)	-2.4
A_{FB}^c	0.0689(35)	0.0742(7)	-1.5
$s_l^2 (Q_{FB})$	0.2321(10)	0.2314(2)	0.7
$s_l^2 (A_{LR})$	0.2310(3)	0.2314(2)	-1.5
A_b	0.911(25)	0.9349(1)	-1.0
A_c	0.630(26)	0.6683(6)	-1.5
m_W [GeV]	80.434(37)	80.397(23)	1.0
$s_W^2 (\nu N)$	0.2255(21)	0.2231(2)	1.1
m_t [GeV]	174.3(5.1)	174.0(4.2)	0.1
m_H [GeV]		55^{+45}_{-26}	
$\hat{\alpha}_s$		0.1183(27)	
$\bar{\alpha}^{-1}$	128.88(9)	128.85(9)	0.3
χ^2/n_{dof}		21.4/14	

$m_E = m_D$. Any value of higgs mass above 113.3 GeV is allowed⁸ in our fits, however χ^2 appears to be minimal for $m_H = 113$ GeV.

In Figure 1 the excluded domains in coordinates $(N_g, \Delta m)$ are shown (here $\Delta m = (m_U^2 - m_D^2)^{1/2}$). Minimum of χ^2 corresponds to $N_g = 0.1$. We see that one extra generation corresponds to 2σ approximately.

We checked that similar bounds are valid for the general choice of heavy masses of leptons and quarks. In particular we found that for $m_N = m_D = 130$ GeV and $m_E = m_U$ one extra generation is excluded at 1.5σ level, while for $m_E = m_U = 130$ GeV and $m_N = m_D$ the limits are even stronger than in Fig. 1. So the extra generations are excluded by the electroweak precision data, if all extra fermions are heavy: $m \gtrsim m_Z$.

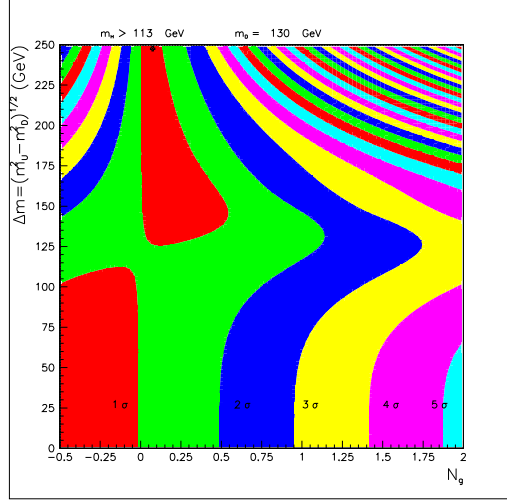


Figure 1. Constraints on the number of extra generations N_g and the mass difference in the extra generations Δm . The lowest allowed value $m_D = 130$ GeV from Tevatron search was used and $m_E = m_D$, $m_N = m_U$ was assumed.

3 Extra generations in case of SUSY

When SUSY particles are heavy they decouple and the same standard model exclusion plots shown in Fig. 1 are valid. One possible exception is a contribution of the third generation squark doublet, enhanced by large stop-sbottom splitting. In this way we get noticeable positive contributions to functions V_i ^{9,10}, which may help to compensate negative contributions of degenerate extra generations. We analyze the simplest case of the absence of $\tilde{t}_L - \tilde{t}_R$ mixing in Fig. 2. In this figure the case of degenerate extra generations with common mass 130 GeV is considered (contributions of superpartners of new generations to V_i are negligible since new up- and down-particles are degenerate). Exclusion plot is presented in coordinates $(N_g, m_{sbottom})$. We see that with inclusion of SUSY new heavy generations are also disfavoured.

Situation changes in case of light chargino and neutralino. The latter are still not excluded - dedicated search at LEP II

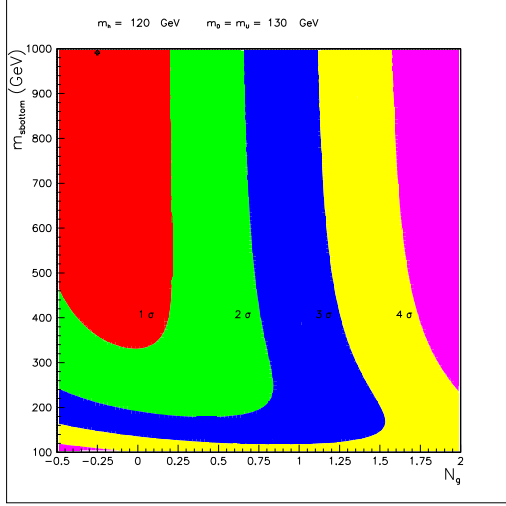


Figure 2. The 2-dimensional exclusion plot for the N_g degenerate extra generations and the mass of sbottom $m_{\tilde{b}}$ in SUSY models and for the choice $m_D = m_U = m_E = m_N = 130$ GeV, using $m_h = 120$ GeV, $m_{\tilde{g}} = 200$ GeV and assuming the absence of $\tilde{t}_L - \tilde{t}_R$ mixing.

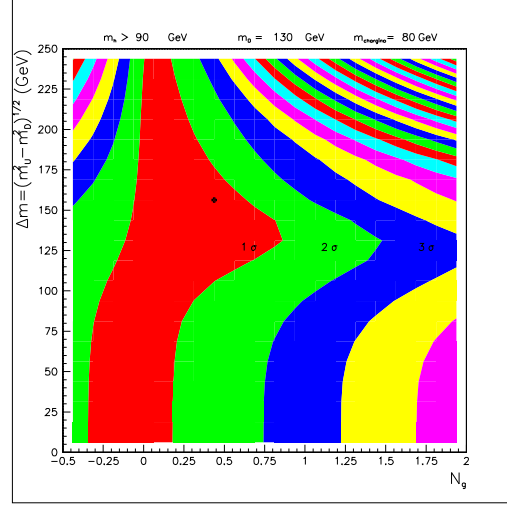


Figure 3. Constraints on the number of extra generations N_g and the mass difference in the extra generations Δm in case of 80 GeV higgsino-dominated quasi degenerate chargino and neutralino. The lowest allowed value $m_D = 130$ GeV from Tevatron search was used and $m_E = m_D$, $m_N = m_U$ was assumed.

still allows the existence of such particles with masses as low as 68 GeV (gaugino region with light sneutrino) ¹¹ or 77 GeV (higgsino case) ¹² if their mass difference is ≈ 1 GeV. Analytical formulas for corrections to the functions V_i from quasi degenerate chargino and neutralino were derived and analyzed in ¹³. Corrections are big and this allows one to get lower bounds on masses of chargino and neutralino: $m_\chi > 54$ GeV for the case of higgsino domination and $m_\chi > 61$ GeV for the case of wino domination at 95% CL.

Fig. 3 demonstrates how presence of chargino-neutralino pair (dominated by higgsino) with mass 80 GeV slightly relaxes the bounds shown on Fig. 1. We see that one extra generation of heavy fermions is allowed within 1.5σ domain in case of the light chargino.

4 Heavy neutrino with $m_N < m_Z$

For particles with masses of the order of $m_Z/2$ oblique corrections drastically differ

from what we have for masses $\gtrsim m_Z$. In particular, renormalization of Z -boson wave function produces large negative contribution to V_A . Quasi-stable neutral lepton N should have the mass slightly above $m_Z/2$ to avoid increasing the invisible Z -width and it should have the mixing angle with three known generations smaller than 10^{-6} to avoid desintegration in the detector. We consider new heavy neutrino with Dirac mass and we suppose that the Majorana mass of N_R is negligible. From the analysis of the initial set of precision data in papers ^{14,15} it was found that the existence of additional light fermions with masses ≈ 50 GeV is allowed. Now analyzing all precision data and using bounds from direct searches we conclude, that the only presently allowed light fermion is neutral lepton N .

As an example we take $m_U = 220$ GeV, $m_D = 200$ GeV, $m_E = 100$ GeV and draw exclusion plot in coordinates (m_N, N_g) , see Fig. 4. From this plot it is clear that for the case of fourth generation with $m_N \approx 50$ GeV

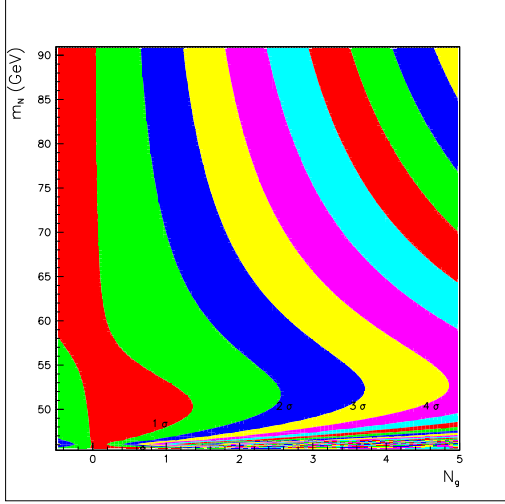


Figure 4. Constraints on the number of extra generations N_g and the mass of the neutral heavy lepton m_N . The values $m_U = 220$ GeV, $m_D = 200$ GeV, $M_E = 100$ GeV were used.

description of the data is not worse than for the Standard Model and that even two new generations with $m_{N_1} \approx m_{N_2} \approx 50$ GeV are allowed within 1.5σ .

5 Possibility for the direct search of the 50 GeV heavy neutrino

The direct search of the heavy neutrino is possible in e^+e^- -annihilation into a pair of heavy neutrinos with the emission of initial state bremsstrahlung photon

$$e^+e^- \rightarrow \gamma + N\bar{N} \quad (1)$$

The main background is the production of the pairs of conventional neutrinos with initial state bremsstrahlung photon

$$e^+e^- \rightarrow \gamma + \nu_i\bar{\nu}_i \quad (2)$$

where $i = e, \mu, \tau$. These background neutrinos are produced in decays of real and virtual Z . In case of $\nu_e\bar{\nu}_e$, two mechanisms contribute, through s -channel Z boson and from t -channel exchange of W boson. We calculated the signal and background distributions and rates ¹⁶ using CompHEP ¹⁷ computer code.

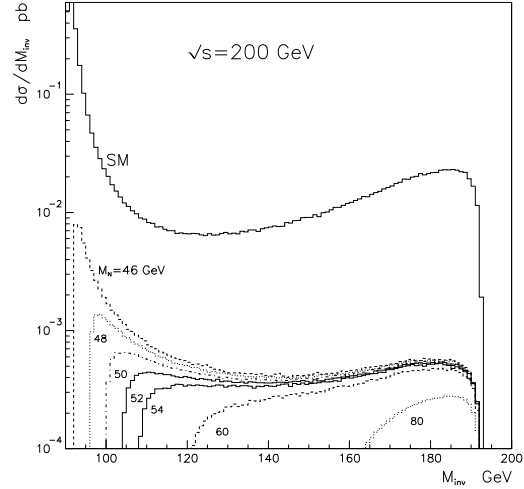


Figure 5. $d\sigma/dM_{inv}$ (in pb) for Standard Model and for the different values of m_N .

In Fig. 5 the distribution on “invisible” mass M_{inv} (invariant mass of the neutrino pair) is represented for SM background and the $N\bar{N}$ signal for $\sqrt{s} = 200$ GeV and different values of N masses, $M_N = 46 - 100$ GeV. Here we applied kinematical cuts on the photon polar angle and transverse momentum, $|\cos\vartheta_\gamma| < 0.95$ and $p_T^\gamma > 0.0375\sqrt{s}$, being the ALEPH selection criteria ¹⁸. The photon detection efficiency 74% is assumed. For highest significance of the $N\bar{N}$ signal, evaluated as $N_S/\sqrt{N_B}$, one should include whole interval on M_{inv} allowed kinematically, so we applied $M_{inv} > 2m_N$ cut.

On Fig. 6 the signal significances are represented as a function of m_N . One can derive that only the analysis based on combined data from all four experiments both from 1997-1999 runs ($\sqrt{s} = 182 - 202$ GeV) and from the current run, in total ~ 2600 pb⁻¹, can exclude at 95% CL the interval of N mass up to ~ 50 GeV.

Another possibility is to search for 50 GeV neutrino at the future TESLA $e^+ - e^-$ electron-positron linear collider. The in-

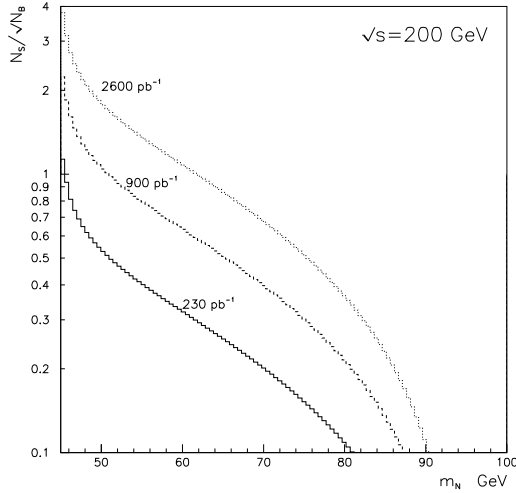


Figure 6. $N\bar{N}$ signal significances at LEP-2 at different statistics as function of the neutrino mass.

crease in energy leads to the decrease both of the signal and the background, but it is compensated by the proposed increase of luminosity of $300 \text{ fb}^{-1}/\text{year}$ ¹⁹. Further advantage of the linear collider is the possibility to use polarized beams. This is important in suppressing the cross section of $e^+e^- \rightarrow \nu_e\bar{\nu}_e\gamma$ as this reaction goes mainly through the t -channel exchange of the W boson. However, even without exploiting the beam polarization the advantage of TESLA in the total number of events is extremely important. Thus, Standard Model is expected to give approximately 0.3 million single photon events for $M_{inv} > 100 \text{ GeV}$ while the number of 50 GeV neutrino pairs would be about 4000. Although the signal over background ratio is still small (2.3-0.5% for $m_N = 45 - 100 \text{ GeV}$ correspondingly) the significance of the signal is excellent, higher than 5 standard deviations for $m_N < 60 \text{ GeV}$.

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1 Guidelines

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The hard copy may be produced using the instructions given in the file *stlread.1st*, which are repeated in this section. You should have three files in total.^a

readme.txt — the preliminary guide.

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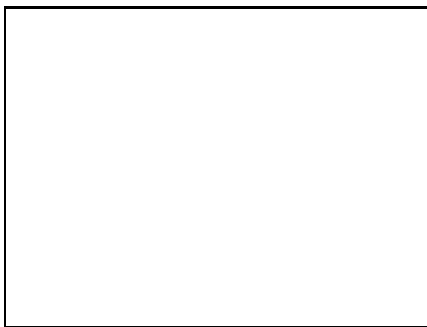


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Paragraphs should have a first line indented by about 0.25in (6mm), except where the paragraph is preceded by a heading, and the abstract should be indented on both sides by 0.25in (6mm) from the main body of the

Table 1. This is a Small Table.

Title	ϵ'	λ	γ
3.5687	3.4567	3.8746	2.8934
Trans Process for Decay		2.8989	4.2928
6.8977	8.9087		

text.

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1.4 Equations

Equations should be confined to one column wherever possible, as in Eq. (2), and the `eqnarray` environment may be used to split equations into several lines, for example in Eq. (3), or to align several equations. An alternative method is given in Eq. (4) for long sets of equations where only one referencing equation number is wanted.

If it's essential to have a two-column wide equation then use the method of Eq. (1) above. The surrounding environment is important here. In the text file `ws-p10x7.tex` make sure that you keep the declarations `\begin{table*}` and `\end{table*}` and only change the equation and its label within the inner equation environment.

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We recommend the use of single column-wide tables wherever possible. For the page wide tables, use the environment given in the example of Table 2. Do not change the latex commands from `\begin{table*}` to `\begin{tabular}`, or from `\end{tabular}` to `\end{table*}`, apart from inserting your own caption heading and table label. The caption heading for a table should be placed at the top of the table.

1.6 Figures

The same arguments apply as are given above for tables, i.e. it is preferable to have figures that fit into one column of the text. If this is not possible, then use the example of figure 2 and use the commands `\begin{figure*}` and `\end{figure*}`. The `\figurebox{...}{...}` command is defined with three arguments. First argument is for the figure height, second argument is for figure width, and the third will be for the actual figure/image file name, (i.e. eps/ps file name).

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1.7 Limitations on the Placement of Equations, Tables and Figures

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Table 2. Experimental Data bearing on $\Gamma(K \rightarrow \pi\pi\gamma)$ for the K_S^0 , K_L^0 and K^- mesons.

Meson	$\Gamma(\pi^+\pi^-) s^{-1}$	$\Gamma(\pi^+\pi^-\gamma) s^{-1}$	
K_S^0	0.769×10^{10}	5.46×10^7	No DE observed, not even (IB)-E1 interference, despite large statistics, for $E_\gamma^* > 20$ MeV.
K_L^0	3.93×10^4	0.90×10^3 (DE = 0.62×10^3)	DE prominent, exceeding IB over the range of measurement $20 < E_\gamma^* < 160$ MeV.
	$\Gamma(\pi^-\pi^0) s^{-1}$	$\Gamma(\pi^-\pi^0\gamma) s^{-1}$	
K^-	1.711×10^7	2.22×10^4 (DE = 1.46×10^3)	No (IB)-E1 interference seen but data shows excess events relative to IB over $E_\gamma^* = 80$ to 100 MeV

will appear on a separate page devoted to figures and tables. Again, we would recommend making any necessary adjustments to the layout of the figures and tables only in the final draft. It is also simplest to sort out line and page breaks in the last stages.

1.8 Footnotes, the Bibliography, Appendices and Acknowledgments

Acknowledgments to funding bodies etc. may be placed in a separate section at the end of the text, before the Appendices. This should not be numbered so use `\section*{Acknowledgments}`.

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Footnotes are denoted by a letter superscript in the text, and references are denoted by a number superscript. We have used `\bibitem` to produce the bibliography. Citations in the text use the labels defined in the bibitem declaration, for example, the first paper by Jarlskog¹ is cited using the command `\cite{ja}`.

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2 Sample Text

The following may be (and has been) described as ‘dangerously irrelevant’ physics. The Lorentz-invariant phase space integral for a general n-body decay from a particle

with momentum P and mass M is given by:

$$I((P-k_i)^2, m_i^2, M) = \frac{1}{(2\pi)^5} \int \frac{d^3 k_i}{2\omega_i} \delta^4(P-k_i). \quad (2)$$

The only experiment on $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ since 1976 is that of Bolotov *et al.*³ The photon spectrum observed certainly exceeds the IB spectrum for $E_\gamma^* \geq 70$ MeV. These authors report definite evidence for DE, however they “conclude that [IB-DE] interference is not observed” in their experiment but such interference is expected to be seen.

Although PDG has recorded the Serphukov experiment as K^+ decay, it is in fact a K^- experiment. This is not a trivial difference. Assuming CPT invariance to be satisfied, CP violation is equivalent with T violation. The latter has to do with phases in the DE processes. Part of the calculated phase will arise from the final-state interactions, but there will also be a non-zero phase in the initial Lagrangian which may feed through to the phases in the final amplitudes. These Lagrangian phases will be different for the K^+ and K^- processes, and can give rise to differences between spectra and rates for K^+ and K^- mesons.

2.1 Parametrizations of the CKM Matrix

It is emphasised that there are two necessary conditions required for any acceptable parametrization of the quark mixing matrix. The first is that the matrix must be unitary, and the second is that it should contain a CP violating phase δ . In Sec. 2.2 the connection between invariants (of form similar to J) and unitarity relations will be examined further for the more general $n \times n$ case. For the present, it's sufficient to note that J is equal to just twice the area of any one of these triangles. This does not mean that a non-zero J follows from unitarity alone; if J equalled zero, the unitarity constraint would still hold, but the triangle would collapse to a straight

line and the measurement of J would be of no use in determining the existence of CP violation if the quark mixing matrix was in fact represented by such a matrix. The reason is that such a matrix is not a faithful representation of the group, i.e. it does not cover all of the parameter space available.

$$\begin{aligned} T = & \text{Im}[V_{11}V_{12}^*V_{21}^*V_{22}] \\ & + \text{Im}[V_{12}V_{13}^*V_{22}^*V_{23}] \\ & - \text{Im}[V_{12}V_{13}^*V_{22}^*V_{23}] \\ & - \text{Im}[V_{33}V_{31}^*V_{13}^*V_{11}]. \end{aligned} \quad (3)$$

There are only 162 quark mixing matrices using these parameters which are to first order in the phase variable $e^{i\delta}$ as is the case for the Jarlskog parametrizations, and for which J is not identically zero.^c It should be emphasised that these are physically identical and form just one true parametrization.

2.2 Four and N-Generation Mixing Matrix

Murnaghan² provided the most general representation of a 4×4 unitary matrix given in Eq. (1). We have calculated the possible combinations and have found that there are eight distinct parametrizations.

The unitary nature of the matrix imposes eight conditions on the connections between adjacent rows and columns, analogous to the six unitarity triangles for the three family case, but for four generations the unitarity condition forms a quadrilateral in the imaginary plane.

We have found only one set of invariants that are independent of their positions in the matrix, i.e. for which one can choose any element to be the ‘starting point’ element $V_{j,\alpha}$ in the definitions of \mathbf{K} , \mathbf{L} and \mathbf{M} given below

^cAn example of a matrix which has elements containing the phase variable $e^{i\delta}$ to second order, i.e. elements with a phase variable $e^{2i\delta}$ is given at the end of this section.



Figure 2. Dynamics of two limit cycle oscillators. The repulsive case ($K < 0$) and attractive case ($K > 0$) are shown. The rate of phase separation, f , is plotted versus the phase separation between the oscillators.

(where the invariants are the sums or differences of the imaginary parts of four plaquettes). These are not however independent of the choice of parametrization but are invariant for each of the individual choices.

$$\begin{aligned}
\mathbf{K} &= \Im[V_{j,\alpha} V_{j,\alpha+1}^* V_{j+1,\alpha}^* V_{j+1,\alpha+1}] \\
&\quad + \Im[V_{k,\alpha+2} V_{k,\alpha+3}^* V_{k+1,\alpha+2}^* V_{k+1,\alpha+3}] \\
&\quad + \Im[V_{j+2,\beta} V_{j+2,\beta+1}^* V_{j+3,\beta}^* V_{j+3,\beta+1}] \\
&\quad + \Im[V_{k+2,\beta+2} V_{k+2,\beta+3}^* V_{k+3,\beta+2}^*] \\
\mathbf{L} &= \Im[V_{j+1,\alpha} V_{j+1,\alpha+1}^* V_{k,\alpha+2}^* V_{k,\alpha+3}] \\
&\quad - \Im[V_{j,\alpha} V_{j,\alpha+1}^* V_{k+1,\alpha+2}^* V_{k+1,\alpha+3}] \\
&\quad + \Im[V_{j+3,\beta} V_{j+3,\beta+1}^* V_{k+2,\beta+2}^* V_{k+2,\beta+3}] \\
&\quad - \Im[V_{j+2,\beta} V_{j+2,\beta+1}^* V_{k+3,\beta+2}^* V_{k+3,\beta+3}] \\
\mathbf{M} &= \Im[V_{j,\alpha+1} V_{j,\alpha+1}^* V_{j+1,\alpha+1}^* V_{j+1,\alpha}] \\
&\quad + \Im[V_{k,\alpha+2} V_{k,\alpha+3}^* V_{k+1,\alpha+2}^* V_{k+1,\alpha+3}] \\
&\quad + \Im[V_{j+2,\beta+1} V_{j+2,\beta+1}^* V_{j+3,\beta+1}^* V_{j+3,\beta}] \\
&\quad + \Im[V_{k+2,\beta+2} V_{k+2,\beta+3}^* V_{k+3,\beta+2}^*], \quad (4)
\end{aligned}$$

where $k = j$ or $j + 1$ and $\beta = \alpha$ or $\alpha + 1$, but

if $k = j + 1$, then $\beta \neq \alpha + 1$ and similarly, if $\beta = \alpha + 1$ then $k \neq j + 1$.

Acknowledgments

This is where one places acknowledgments for funding bodies etc. Note that there are no section numbers for the Acknowledgments and Appendix. The style file will automatically generate the heading for references.

Appendix

We can insert an appendix here and place equations so that they are given numbers such as Eq. (5).

$$x = y. \quad (5)$$

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3. J.D. Bjorken and I. Duniety, *Phys. Rev. D* **36**, 2109 (1987).

